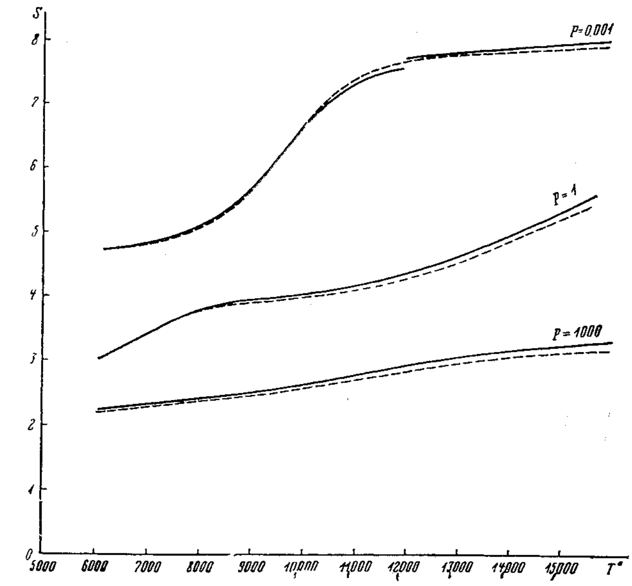

Fig. 5.

In the foregoing expressions

$$\alpha = \alpha_1 + \alpha_2$$

$$L_i = \frac{\alpha_i(\eta_i - \alpha_i)}{2\eta_i - \alpha_i}$$

$$M_i = D^i/T \qquad i = 1; 2$$

where P is expressed in physical atmospheres.

ρ = g/cm³
T = °K
D_1 = 59,369°K,† the dissociation energy of a molecule of oxygen, referred to Boltzmann's constant (using 5.1155 ev/molecule)
D_2 = 113,262,† the dissociation energy of a molecule of nitrogen referred to Boltzmann's constant (k = 1.38042×10^{-16} erg deg^{-1}, using 9.7592 ev/molecule)
I = 169,924°K
η_1 = 0.21, the molecular fraction of oxygen in air under normal conditions
η_2 = 0.79, the molecular fraction of nitrogen in air under normal conditions. B is assumed equal to $9 \cdot 10^{-1}$

$$R = 0.06874 \frac{\text{g} \cdot \text{cal}}{\text{g} \cdot \text{deg}} = 2.838 \frac{\text{atm} \cdot \text{cm}^3}{\text{g} \cdot \text{deg}} = 287.5 \frac{m^2}{\text{sec}^2 \cdot \text{deg}}$$

† The values for D_1 and D_2 appearing in the text have been recommended by the reviewer. The values used in the original Russian paper are: D_1 = 59,400; D_2 = 113,300—Editor.

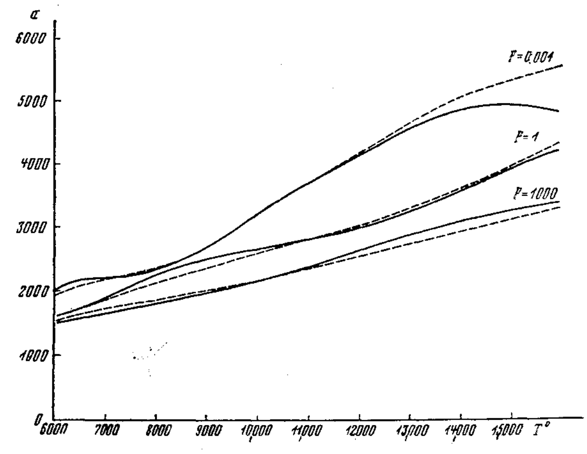

Fig. 6.

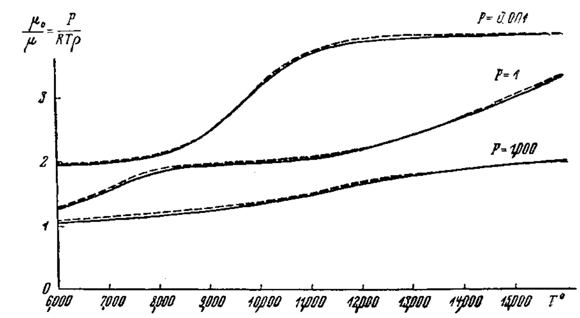

Fig. 7.

The parameter ν entering into the expressions in Ref. 1 is put equal to zero.

The results of computations based on the combined formulas are shown in Figs. 1–7. The solid lines represent exact values of the thermodynamic functions, the broken lines those obtained by means of the approximate formulas.

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Reviewer's Comment

The thermodynamic data presented by the author were computed on the assumptions that the gases are ideal and that the mixture is in chemical equilibrium. Standard thermodynamic procedures have been used in computing the data.

The substitution of the average energy (expressed in °K) of a singly ionized particle for the principal ions in air, represents an interesting feature. This is permissible because of closeness of the ionization potentials of nitrogen and oxygen atoms (14.5482 and 13.6173 ev, respectively).[1] Hansen[2] discussed this possibility earlier.

Using the accepted values for the ionization energy for oxygen atoms, the corresponding temperature was found to be 158,038°K instead of 157,036°K as reported by the author. The temperatures corresponding to the ionization energies of nitrogen and argon were found to be almost identical to the values reported (168,842°K and 182,891.9°K, respectively). The average value of $I = i_A/k$ in degrees K turns out to be 169,924 and not 166,500 as reported. These minor errors apparently do not affect the data significantly. A comparison of the specific heat data with similar data published by American scientists[2–4] show reasonably good agreement.

It should be pointed out that in the investigations of hypersonic phenomena, the specific heat and other thermodynamic quantities of the pure species are the most useful. Much data of this type, which covers a wide range of temperatures, are available.[1,2,5–7]

The author does not list the following input data: a) electronic degeneracy of the states, b) the number of electronic states used, and c) the standard density or the standard temperature employed in the calculations. It is presumed that such information is covered adequately in Russian references cited.

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Hypersonic Area Rule

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In an earlier work[1] on the hypothesis[2] that the whole mass of gas is concentrated in an infinitely thin layer contiguous to the shock wave, a hypersonic area rule was formulated. According to this rule, when there is a flow past thin blunted nonaxisymmetrical bodies which have equal quantities of bluntness resistance and the same rules of variation in the direction of flow of the cross-sectional areas, and of the surface of the shock waves, the rules of pressure change and consequently the forces of resistance acting on the body as well, coincide, in which case the surfaces of the shock waves have an axial symmetry.

In the present work the limits of applicability of the results of Ref. 1 are established, and the hypersonic area rule is made more accurate by means of the introduction of an entropy layer.

1. Determination of the Limits of Applicability of the Results of Ref. 1

As an example of the application of the hypersonic area rule,[1] we construct a body equivalent to a thin round cone, i.e., one having the same quantity of bluntness resistance as a cone and the same trend of change in cross-sectional area in the direction of flow. The cross section of the body is postulated as having the shape of an ellipse, the major semiaxis of which is equal to the radius of the shock wave, and the area equal to the area of the cross section of the round cone (Fig. 1). In other words, the eccentricity of the ellipse in each section has its maximum possible value compatible with the requirement (condition 3 of Ref. 1) according to which the body should not go beyond the limits of a volume confined to the surface of the shock wave.

As was mentioned in the earlier work,[1] the area rule may be combined with the similarity law when the flow occurs past thin blunted bodies,[2] as a result of which the dimensionless quantities characterizing the flow are determined, for a fixed value of the adiabatic index $\varkappa$, by two dimensionless parameters: the known parameter of similarity when the flow is past thin blunt bodies $K = M_\infty\tau$ and the parameter $K_1 = (\pi/2c_xS)^{1/2}L\tau^2$, characterizing the influence of the bluntness, which in its order of magnitude is equal to the square root of the ratio of the resistance of the body to the resistance of the bluntness. Here $\tau = S^{1/2}/L$ is the small dimensionless quantity characterizing the thickness of the body; S some characteristic cross-sectional area of the body; L the length of the body; c_x and S, respectively, are the coefficient of bluntness resistance and the midships area of the bluntness.

Under the hypothesis that the action of bluntness may be replaced by the effect of an explosion in the leading point of the body with an energy equal to the bluntness resistance, the shape of the bluntness is nonessential.[1] In view of this, the bluntness area is introduced in the expression for k_1 instead[2] of its diameter. Let us assume that the number M_∞ of unperturbed flow equals infinity. Then for a fixed $\varkappa$ the dimensionless variables will depend on the single parameter K_1.

In Fig. 2 are shown (for the case of $\varkappa = 1.4$) the relations of the quantities k (ratio of major to minor semiaxis of the ellipse) and $(X - X_0)/X_0$ (the ratio of the resistance of the body, after the deduction of the bluntness resistance, to the bluntness resistance) in function $K_1 = (\pi/2c_xS)^{1/2}L \tan^2 \alpha$, where α is the angle of the half-aperture of the round cone. The shape of the shock wave was determined from the solution of the problem of a flow past a thin blunt cone according to Ref. 2. The area rule has a significance in use where $X/X_0 \geqq 1.1$, corresponding (see Fig. 2) to $K_1 \geq 0.1$. For low values of K_1 the resistance of the body is practically determined by the magnitude of the bluntness resistance. For large values of K_1 the area rule loses its force when k approaches unity, or more accurately[1] when $k - 1 \sim (\varkappa - 1)/(\varkappa + 1)$, which takes place approximately when $K_1 = 1.2$.

Thus the range of applicability of the area rule falls within the limits of $0.1 \leqq K_1 \leqq 1.2$. Here the ellipse in the cross section of the body may have a fairly elongated shape, different from a circle ($13 \geqq k \geqq 1.3$). This result may be of practical interest. However, in view of the fact that the results of Ref. 1 are obtained under rough assumptions of the concentration of the whole mass of gas in an infinitely thin

Translated from Inzhenernyi Zhurnal (Engineering Journal) 1, No. 1, 159–163 (1961). Translated by Singer, Smith, and Co., New York.